Evolution of a scroll ring in an oscillatory medium close to a Neumann boundary

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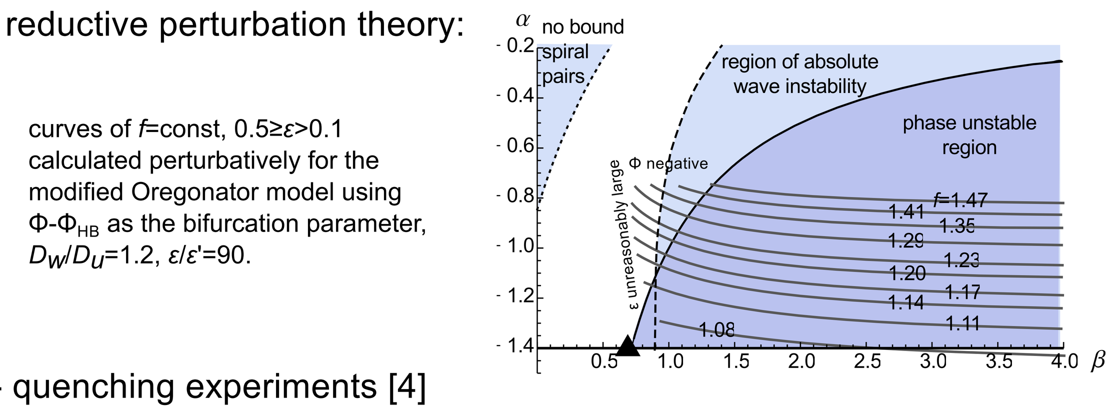
Oscillatory media

Close to a supercritical Hopf bifurcation, reaction-diffusion systems can be approximated by the complex Ginzburg-Landau equation (CGLE) [1].

$$\frac{\partial}{\partial t}A = A + (1+i\alpha)\Delta A - (1+i\beta)|A|^2 A$$

parameters α and β can be obtained by:

- reductive perturbation theory:



- quenching experiments [4]
- measurement of filament tension in units adapted to the system:

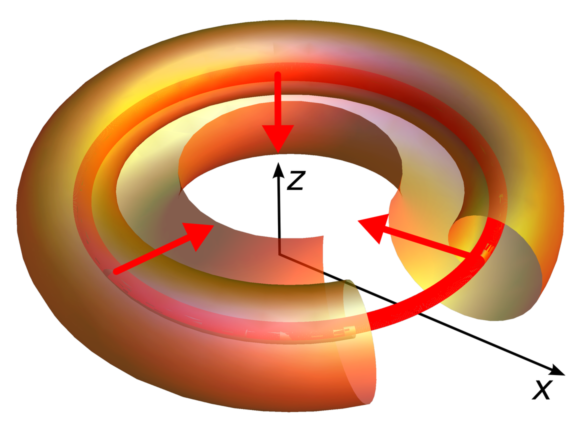
$$\frac{1+\alpha^2}{\alpha-\beta} = \frac{Ck^2}{\omega(k)-\omega_{hom}}$$

C - filament tension determined experimentally from $-\dot{R}R$
k, $\omega(k)$ - wave number and respective frequency of a plane wave
ω_{hom} - oscillation frequency of the homogeneous system

Q: What insight can we gain into the 3-D dynamics of RD systems?

Scroll rings

- the filament is a circle of radius $R(t)$



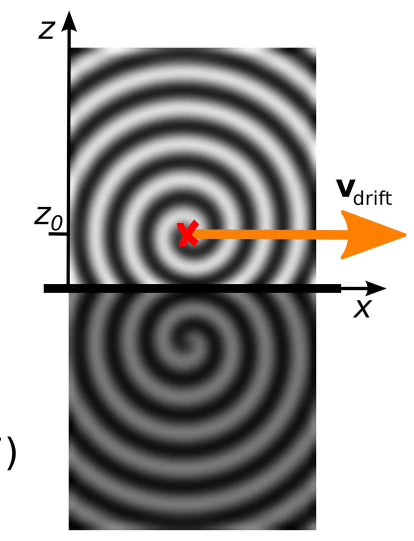
- in the parameter region where a straight filament is stable, scroll rings contract and the drift velocity along their symmetry axis is vanishingly small [2]

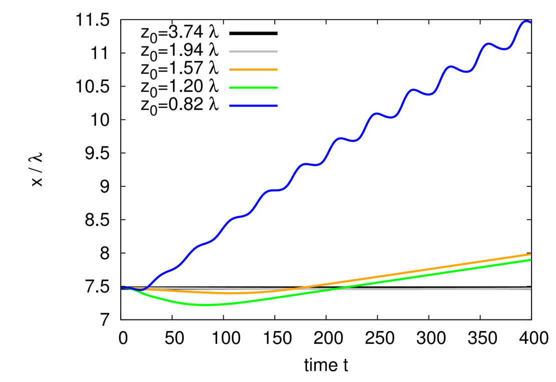
$$\frac{\mathrm{d}}{\mathrm{d}t}R = -\frac{1+\alpha^2}{R} \quad (1) \qquad \frac{\mathrm{d}}{\mathrm{d}t}z = 0 \quad (2)$$

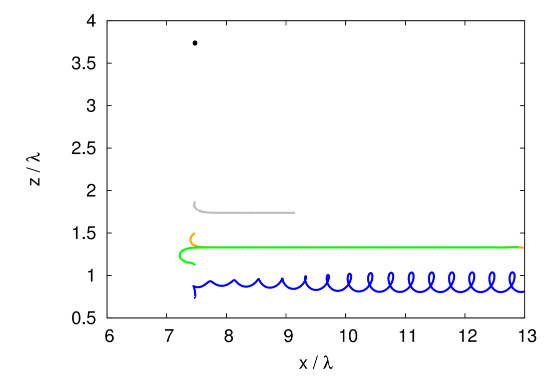
Q: What can we learn about scroll rings in the CGLE?

Boundary induced drift

- interaction of a 2-D spiral wave with a straight Neumann boundary ≙ interaction with its mirror image
- discrete set of attracting trajectories at distances $(z_i^*)_{i\in\mathbb{N}}$ from the boundary [3]
- drift velocity decreases with increasing z_i^*
- i=1: limit cycle motion in a drifting, comoving frame (loop-like attractor [5], α=-1.4, β=0.7)
- no interaction at z>>λ





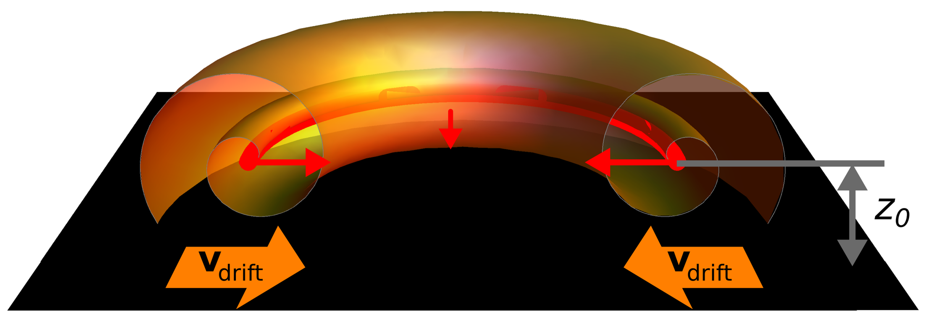


λ: asymptotic wavelength of the unperturbed spiral

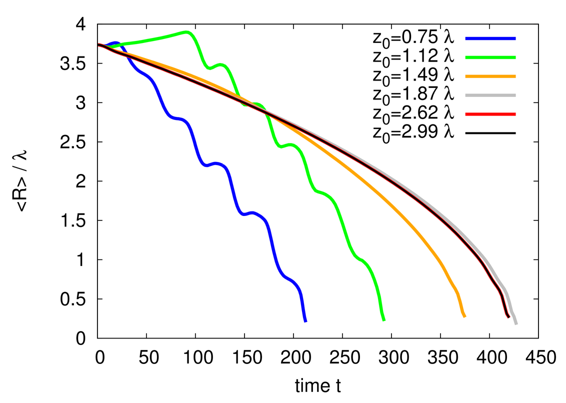
Q: Which effects persist in 3-D?

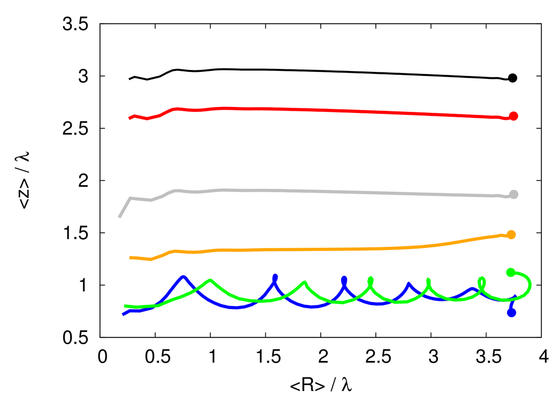
Characterization of 3-D dynamics

Cooperative setting



The intrinsic dynamics of the scroll ring acts in the same direction as the boundary-induced drift.

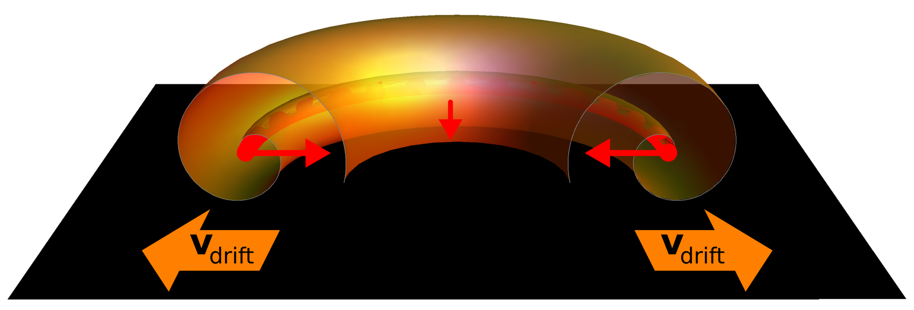




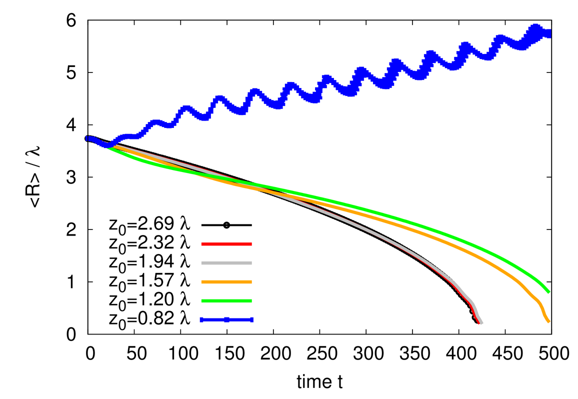
Filament contraction
Depending on the initial distance z_0, three types of behavior are observed:

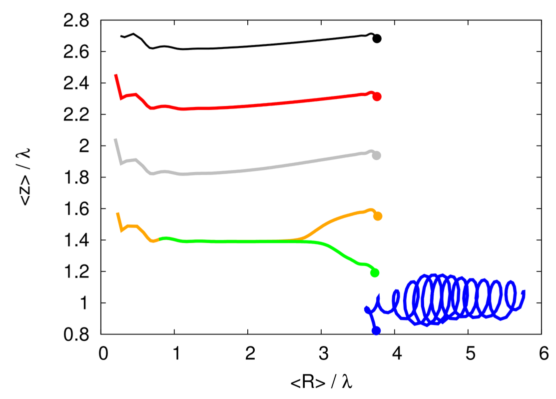
- z_0>1.7λ: the dynamic follows equations (1) and (2) for the free scroll ring
- 1.7λ>z_0>1.2λ: contraction is speeded up
- 1.2λ>z_0: contraction is speeded up, superimposed by an oscillation

Antagonistic setting



The intrinsic dynamics of the scroll ring acts against the boundary-induced drift.





Filament contraction or expansion

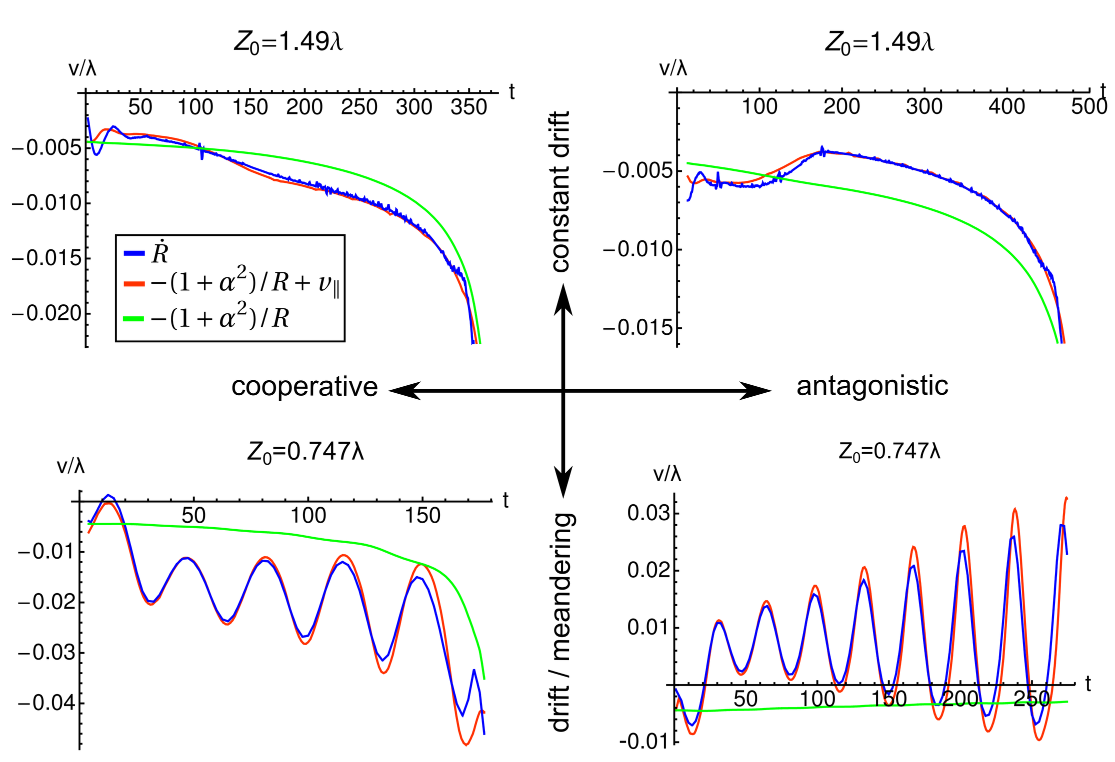
- z_0>1.7λ: free scroll ring approximation remains valid
- 1.7λ>z_0>1.0λ: contraction is slowed down
- 1.0λ>z_0: boundary-induced drift dominates over contraction, resulting in expansion

Phenomenological theory

addition of velocity contibutions

motion due to filament tension
\+ velocity of boundary induced drift

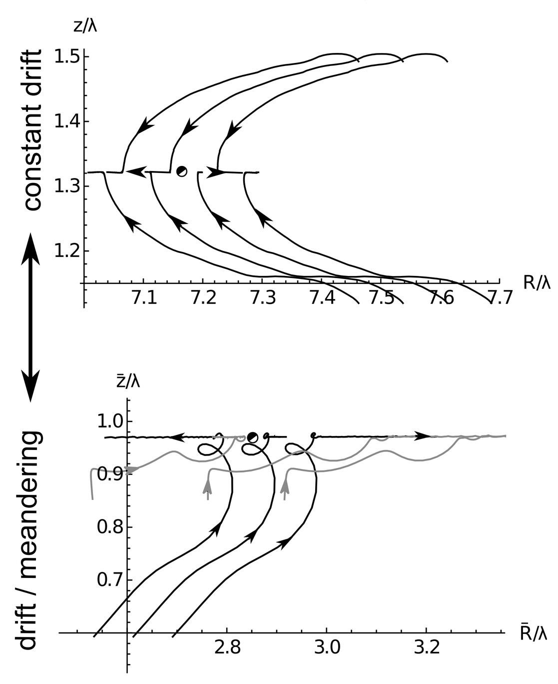
$$\frac{\mathrm{d}}{\mathrm{d}t}R = -\frac{1+\alpha^2}{R} + v_\parallel$$



phase plane

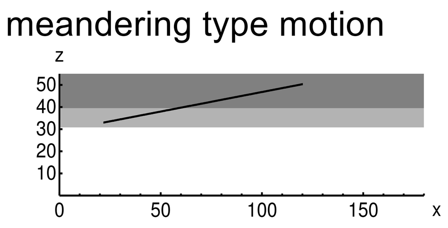
sequence of saddle points (R_i^*, z_i^*) where z_i^* are fixed points from 2-D calculations [3] and

$$R_i^* = \frac{1+\alpha^2}{v_\parallel(z_i^*)}$$

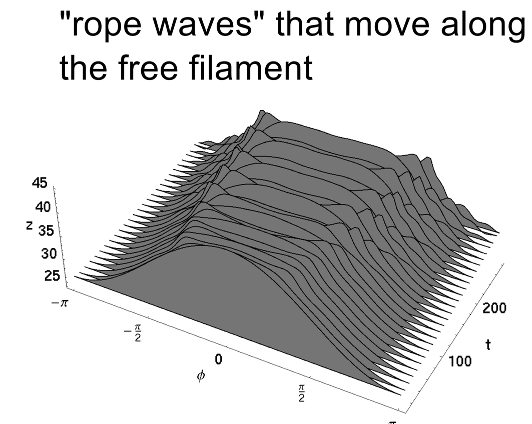


Tilted scroll rings

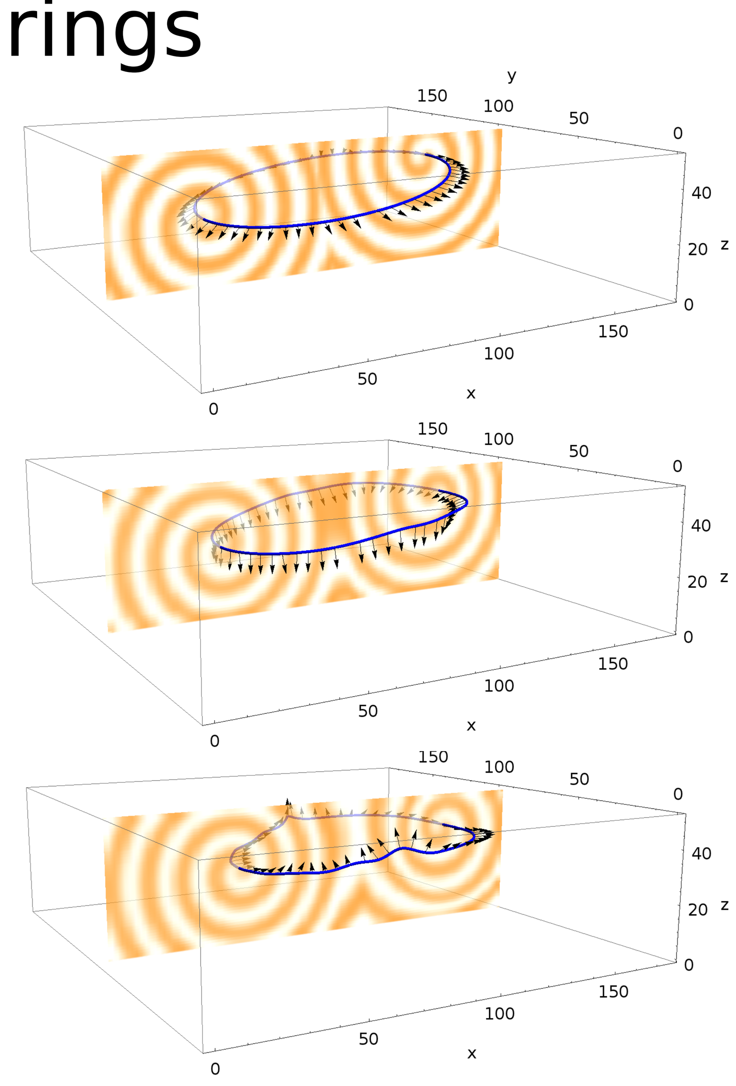
- antagonistic configuration
- a circular segment of the filament belongs to the basin of the attractor that features meandering type motion



- the meandering part of the filament acts as a source of "rope waves" that move along the free filament



- different rotation frequencies result in phase twist



Conclusions

Intrinsic dynamics of scroll ring solutions to the CGLE as given by eqs. (1) and (2) can be strongly modified by the short-range interaction with a Neumann boundary. In particular, contraction can be replaced by expansion, despite positive filament tension.
Boundary-induced drift, as studied in detail for two-dimensional spiral waves [3], is responsible for the observed changes in growth rate. Adding velocity contributions from drift and from intrinsic dynamics yields a growth rate, that is in good agreement with numerical simulations.
When the symmetry axis is inclined with respect to the Neumann boundary and if the dominant dynamic is superimposed drift and meandering, the meandering part of the filament generates waves that run along the free filament. Different rotation frequencies between the meandering part and the free part result in a phase twist.

References

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