Numerical Evaluation of Nonlinear Corrections to Nuclear Parton Evolution

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Inelastic Electron-Proton Scattering

Idea: Break protons up into their constituent particles
by scattering high-energy electrons off them

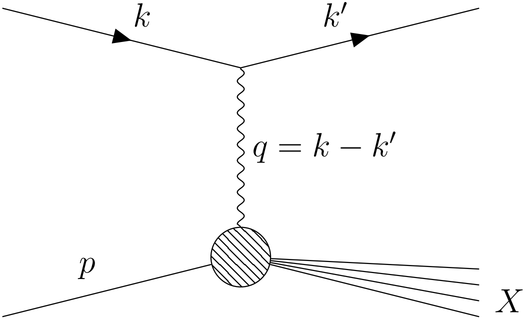
$\to$ cross section:

$$\mathrm{d}\sigma = \frac{1}{F}|\mathcal{M}|^2 \mathrm{d}Q$$

$F \mathrel{\widehat{=}}$ incoming flux

$|\mathcal{M}|^2 \mathrel{\widehat{=}} |i\rangle \to |f\rangle$ rate

$\mathrm{d}Q \mathrel{\widehat{=}} |f\rangle$ phase space



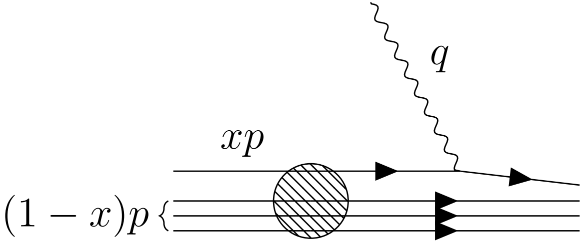
$$\text{Feynman rules} \implies \mathrm{d}\sigma = \frac{\pi M}{(k \cdot p)} \frac{e^4}{q^4} L^{\mu\nu} W_{\mu\nu}\ \Pi^3 k' \quad \text{with}$$

$$W_{\mu\nu} = \frac{F_1}{M}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{F_2}{\nu M^2}\left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right)$$

Parton Model

Assumption: The proton consists of non-interacting quarks.

$\rightarrow$ parton distribution functions (PDFs) $q_i(x)$



$$q_i(x) \hat{=} \text{ probability quark } i \text{ has momentum } xp$$

If quarks are fermions, the cross section follows from QED.

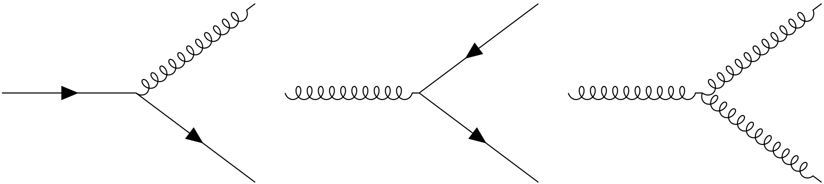
$\Longrightarrow$ Callan-Gross relation: $\boxed{2xF_1(x) = F_2(x) = x \sum_{i=1}^{2N_F} e_i^2 q_i(x)}$

Quantum Chromodynamics

QCD is the theory of the interaction between quarks and gluons.

$$\mathscr{L}_{\mathrm{QCD}} = \overline{\Psi}_f (i\not{D} - m_f)\Psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Processes affecting the PDFs:



q → g emission   g → q emission   g → g emission

→ PDFs now depend on the energy scale $Q^2 = -q^2$

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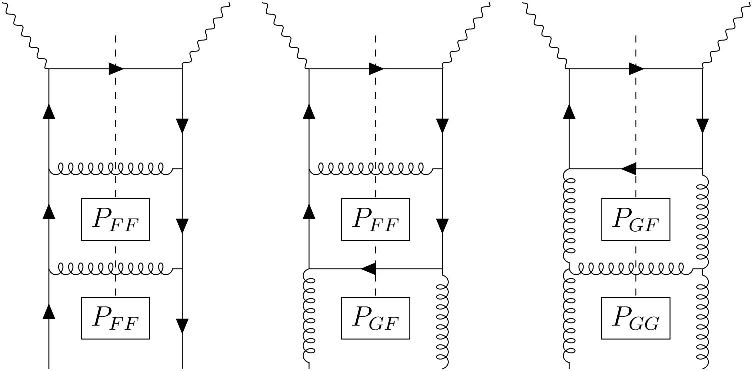
DGLAP Equations

The DGLAP equations describe the Q^2 dependence of the PDFs.



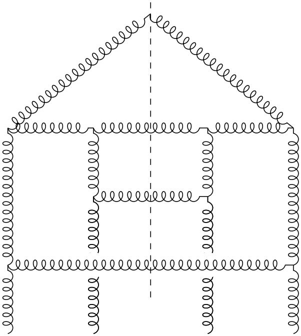
$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix}\Omega(x,Q^2)\\ G(x,Q^2)\end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z^2}x\begin{pmatrix}P_{FF}(x/z) & P_{FG}(x/z)\\ P_{GF}(x/z) & P_{GG}(x/z)\end{pmatrix}\begin{pmatrix}\Omega(z,Q^2)\\ G(z,Q^2)\end{pmatrix}$$

with the momentum densities $\Omega = x\Sigma_f(q_f + \overline{q}_f), G = xg$

GLR-MQ Equations

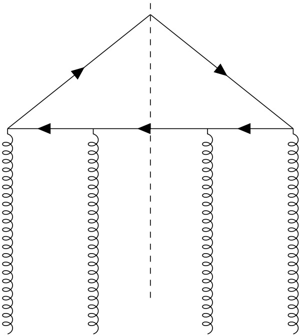
Taking gluon recombination into account modifies the evolution.

gluon-gluon recombination:



$$-\frac{81}{16}\frac{\alpha_s^2}{R^2Q^2}\int_x^1\frac{\mathrm{d}z}{z}G^2(z,Q^2)$$

gluon-quark recombination:



$$-\frac{27}{160}\frac{\alpha_s^2}{R^2Q^2}G^2(x,Q^2)$$

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Convolution Integrals

The evolution is done on a grid in x and Q^2, with $D(x_c, Q_r^2) = D_{rc}$.

$$\int_{x_c}^{1} \mathrm{d}z\, P_{AB}\left(\frac{x_c}{z}\right) D(z) \xrightarrow[\text{Interpolation}]{\text{Linear}} \sum_{k=c}^{n} w_{AB}(x_k, x_c) D_{rk}, \text{ where}$$

$$w_{AB} \sim \int \mathrm{d}z\, P_{AB}(z)$$

P_{AB} can be written as a perturbation series in α_s:

$$P_{AB}(x) = P_{AB}^{(0)}(x) + \frac{\alpha_s}{2\pi} P_{AB}^{(1)}(x) + \dots$$

Leading order contribution:

$$P_{GG}^{(0)}(z) = 2C_G \left[\frac{1}{1-z} + \frac{1}{z} - 2 + z - z^2 \right.$$
$$\left. - \frac{1}{z}\delta(1-z) \int_0^1 \mathrm{d}y\, y \left\{ \frac{1}{1-y} + \frac{1}{y} - 2 + y - y^2 \right\} \right] - \delta(1-z)\frac{2}{3} T_R N_F$$

Convolution Integrals

Next-to-leading order contribution:

$$
\begin{aligned}
P_{GG}^{(1)} = {} & C_F T_R N_F \left[-16 + 8z + \frac{20}{3}z^2 + \frac{4}{3}\cdot\frac{1}{z} - (6+10z)\ln z - 2(1+z)\ln^2 z\right] \\
& + C_G T_R N_F \left[2 - 2z + \frac{26}{9}\left(z^2 - \frac{1}{z}\right) - \frac{4}{3}(1+z)\ln x - \frac{20}{9}\left(\frac{1}{1-z} + \frac{1}{z} - 2 + z - z^2\right)\right] \\
& + C_G^2 \left[\frac{27}{2}(1-z) + \frac{67}{9}\left(z^2 - \frac{1}{z}\right) + \left(-\frac{25}{3} + \frac{11}{3}z - \frac{44}{3}z^2\right)\ln z + 4(1+z)\ln^2 z\right. \\
& \left. + \left(\frac{67}{9} - \frac{\pi^2}{3} - 4\ln z\ln(1-z) + \ln^2 z\right)\left(\frac{1}{1-z} + \frac{1}{z} - 2 + z - z^2\right) + 2\left(\frac{1}{1+z} - \frac{1}{z} - 2 - z - z^2\right)\int\limits_{z/(1+z)}^{1/(1+z)} \frac{\mathrm{d}x}{x}\ln\left(\frac{1-x}{x}\right)\right] \\
& - \frac{1}{z}\delta(1-z)\int\limits_0^1 \mathrm{d}y\, y \left\{ C_F T_R N_F \left[-16 + 8y + \frac{20}{3}y^2 + \frac{4}{3}\cdot\frac{1}{y} - (6+10y)\ln y - 2(1+y)\ln^2 y\right]\right. \\
& + C_G T_R N_F \left[2 - 2y + \frac{26}{9}\left(y^2 - \frac{1}{y}\right) - \frac{4}{3}(1+y)\ln x - \frac{20}{9}\left(\frac{1}{1-y} + \frac{1}{y} - 2 + y - y^2\right)\right] \\
& + C_G^2 \left[\frac{27}{2}(1-y) + \frac{67}{9}\left(y^2 - \frac{1}{y}\right) + \left(-\frac{25}{3} + \frac{11}{3}y - \frac{44}{3}y^2\right)\ln y + 4(1+y)\ln^2 y\right. \\
& \left.\left. + \left(\frac{67}{9} - \frac{\pi^2}{3} - 4\ln y\ln(1-y) + \ln^2 y\right)\left(\frac{1}{1-y} + \frac{1}{y} - 2 + y - y^2\right) + 2\left(\frac{1}{1+y} - \frac{1}{y} - 2 - y - y^2\right)\int\limits_{y/(1+z)}^{1/(1+y)} \frac{\mathrm{d}x}{x}\ln\left(\frac{1-x}{x}\right)\right]\right\} \\
& - \delta(1-z)\int\limits_0^1 \mathrm{d}y\, y \left\{ C_F T_R N_F \left[4 - 9y + (-1+4y)\ln y + (-1+2y)\ln^2 y + 4\ln(1-y)\right.\right. \\
& \left. + \left(10 - \frac{2}{3}\pi^2 + 4\ln y - 4\ln y\ln(1-y) + 2\ln^2 y - 4\ln(1-y) + 2\ln^2(1-y)\right)\left(y^2 + (1-y)^2\right)\right] \\
& + C_G T_R N_F \left[\frac{182}{9} + \frac{14}{9}y + \frac{40}{9}\cdot\frac{1}{y} + \left(\frac{136}{3}y - \frac{38}{3}\right)\ln y - 4\ln(1-y) - (2+8y)\ln^2 y\right. \\
& \left.\left. + \left(\frac{\pi^2}{3} - \frac{218}{9} + \frac{44}{3}\ln y + 4\ln(1-y) - \ln^2 y - 2\ln^2(1-y)\right)\left(y^2 + (1-y)^2\right) + 2\left(y^2 + (1+y)^2\right)\int\limits_{y/(1+y)}^{1/(1+y)} \frac{\mathrm{d}x}{x}\ln\left(\frac{1-x}{x}\right)\right]\right\}
\end{aligned}
$$

Evolution Process

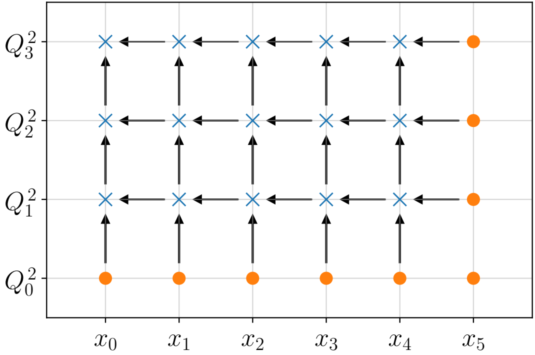
Evolution equations:

$$\Omega'_{rc} = W_{FF}\Omega_{rc} + W_{FG}G_{rc} + M_F - V_1 G_{rc}^2$$

$$G'_{rc} = W_{GF}\Omega_{rc} + W_{GG}G_{rc} + M_G - \left(V_2 G_{rc}^2 + V_3 G_{rc} + N_G\right)$$

Two extra equations follow from a quadratic interpolation:

$$D_{rc} = D_{(r-1)c} + \frac{\Delta_r}{2}\left(D'_{(r-1)c} + D'_{rc}\right)$$



Orange dots:
Starting values

At each point, the four equations are solved for D_{rc} & D'_{rc}

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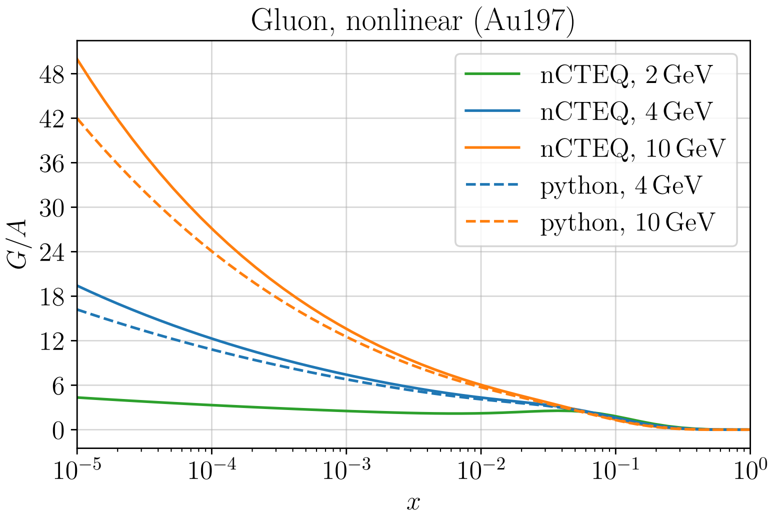
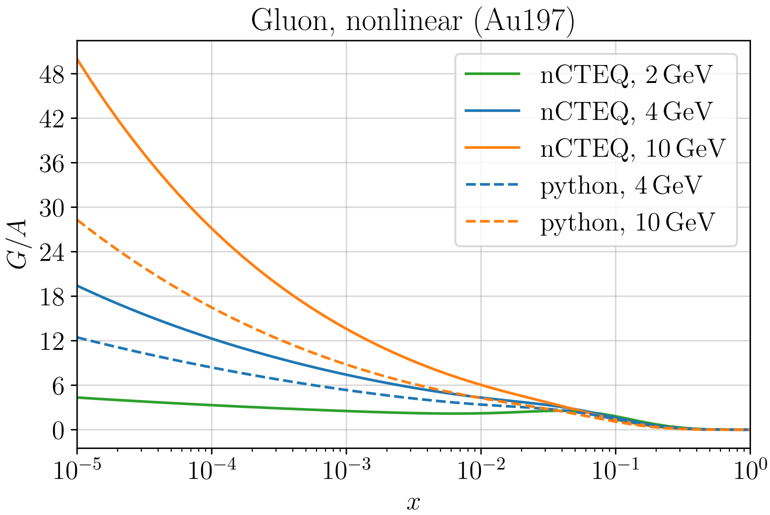
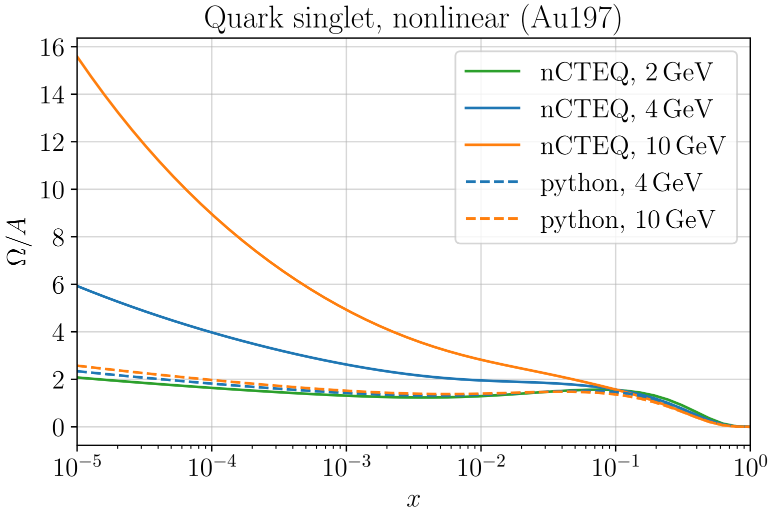
Interpretation



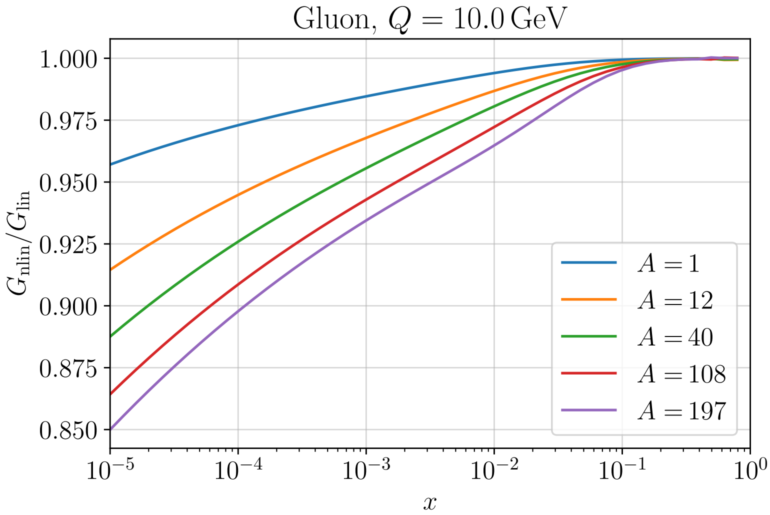
Parton Mixing



Parton Mixing



A Dependence



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